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Importing and Fitting Data Module

You assignment is to import a data set and fit the data set. You can use the examples below to guide you. More advanced fitting approaches are provided below. The data sets you will import and fit are the following:
1. Data 1 - no formula.xls (hint: polynomial)
2. Data 2 - no formula.xls (hint: exponential)
3. transcendental

For each equation, calculate the Residual of Sum of Squares for the fit. Output your best fit and the Residual of Sum of Squares.

Fitting Data that is:
Generated or Imported

- **Example: Importing and fitting data of the form:**

  y=5+5 x^2

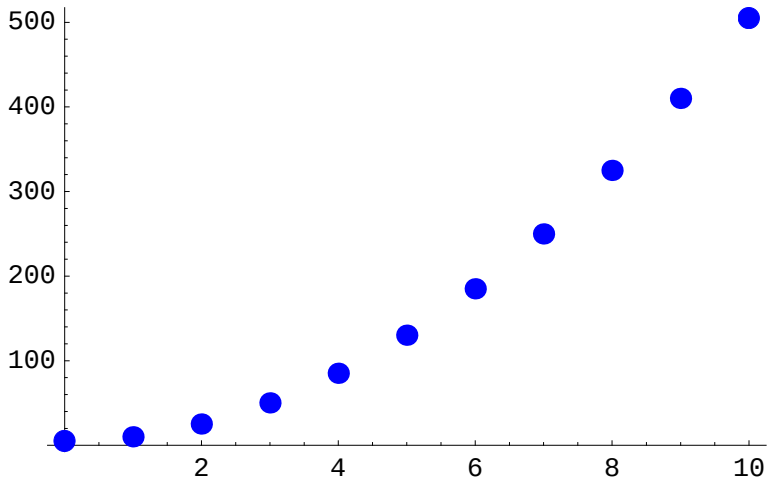
This example uses the following commands:

Import[ ] - this function imports data from several types of sources. I've used an Excel file from which to import data.

FindFit[ ] - this function takes the imported data and fits it to a function I've provided. Note that each constant is constrained to a range of values.

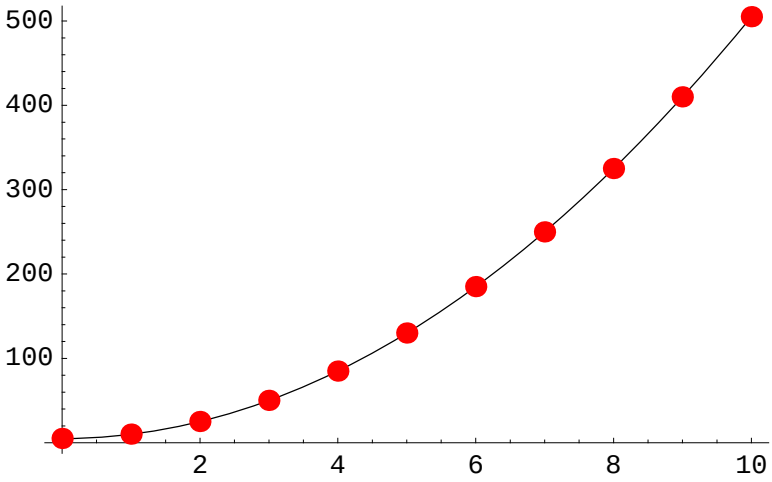
In[141]:=
```
Clear[importdata1, a, b, n, x, fitfunc1]

importdata1 =
 Import["C:\\D drive\\Mathematica&Endnote\\Mathematica\Data\\
    Importdata1.xls"]
ListPlot[importdata1,
 PlotStyle -> {RGBColor[0, 0, 1], AbsolutePointSize[8]}]

fitfunc1[x_] =
 a + b x^n /. FindFit[importdata1, a + b x^n, {{a, 3, 5}, {b, 3, 7}, {n, 1, 2}},
   x]

plotfit1 = Plot[fitfunc1[x], {x, 0, 10}, Epilog -> {
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ importdata1}]
(*Epilog is an option for graphics functions which gives a list
  of graphics primitives to be rendered or added after the main
  part of the graphics is rendered.*)
(*Map[f, expr] or f /@ expr applies f to each element on the first
  level in expr*)

Null
```

Out[142]=
```
{{0., 5.}, {1., 10.}, {2., 25.}, {3., 50.}, {4., 85.}, {5., 130.},
 {6., 185.}, {7., 250.}, {8., 325.}, {9., 410.}, {10., 505.}}
```



Out[143]=
```
- Graphics -
```

Out[144]=
```
5. + 5. x^2.
```



Out[145]=
```
- Graphics -
```

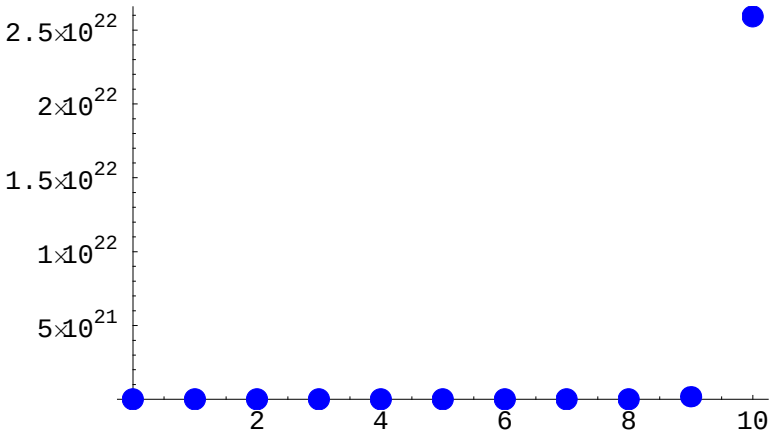
■ Importing a fitting data using an extended FindFit to fit the equation:

y=5+5 Exp[5 x]

Note that I am trying various equations to fit the data and the one below is not so successful. Here, the fitting equation I use is: y = a+b Exp[c x]

In[11]:=
```
Clear[fitfunc2, importdata2, a, b, c, x, plotfit2]

importdata2 =
 Import["C:\\D drive\\Mathematica&Endnote\\Mathematica\Data\\
    Importdata2.xls"]
ListPlot[importdata2,
 PlotStyle -> {RGBColor[0, 0, 1], AbsolutePointSize[8]}]

fitfunc2[x_] =
 a + b Exp[c x] /. FindFit[importdata2, a + b Exp[c x],
   {{a, 4, 5}, {b, 4, 5}, {c, 4, 5}}, x]

plotfit2 = Plot[fitfunc2[x], {x, 0, 10}, Epilog -> {h
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ importdata2}]
```

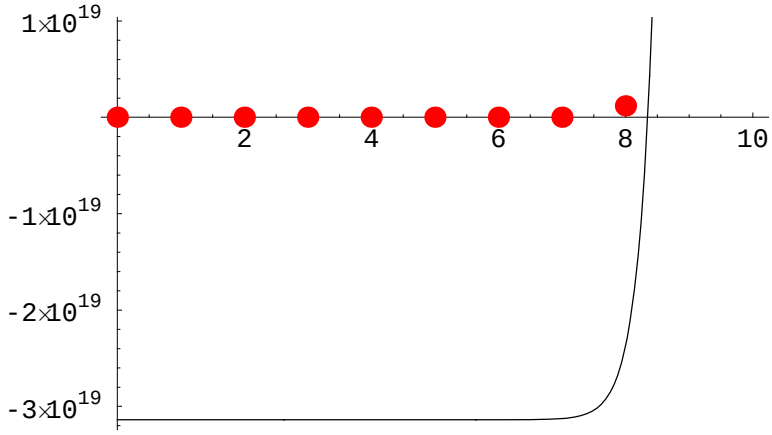
Out[12]=
```
{{0., 10.}, {1., 747.066}, {2., 110137.},
 {3., 1.63451×10^7}, {4., 2.42583×10^9}, {5., 3.60024×10^11},
 {6., 5.34324×10^13}, {7., 7.93007×10^15}, {8., 1.17693×10^18},
 {9., 1.74671×10^20}, {10., 2.59235×10^22}}
```



Out[13]= - Graphics -

FindFit::fmmp : Machine precision is insufficient
to achieve the requested accuracy or precision. More…

Out[14]=
$$-3.13928\times10^{19}+58514.5\,e^{4.06337\,x}$$



Out[15]= - Graphics -

Obviously, the fitting equation does not work, so we need to try another.

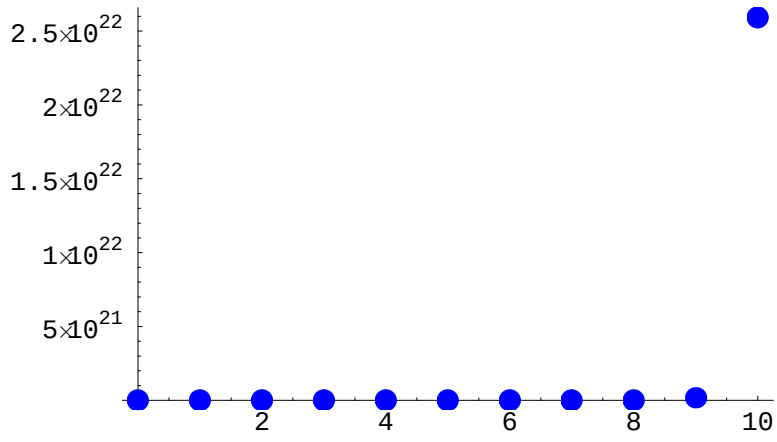
■ Importing a fitting data using an extended FindFit to fit the equation:

y=5+5 Exp[5 x]

The one below is not at all successful. The fitting equation I use is: y = a+a Exp[a x]

In[6]:=
```
Clear[fitfunc2, importdata2, a, b, c, x, plotfit2]

importdata2 =
 Import["C:\\D drive\\Mathematica&Endnote\\Mathematica\Data\\
    Importdata2.xls"]
ListPlot[importdata2,
 PlotStyle -> {RGBColor[0, 0, 1], AbsolutePointSize[8]}]

fitfunc2[x_] =
 a + a Exp[a x] /. FindFit[importdata2, a + a Exp[a x], {a, 4, 5}, x]

plotfit2 = Plot[fitfunc2[x], {x, 0, 10}, Epilog -> {
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ importdata2}]
```

Out[7]=
```
{{0., 10.}, {1., 747.066}, {2., 110137.},
 {3., 1.63451×10^7}, {4., 2.42583×10^9}, {5., 3.60024×10^11},
 {6., 5.34324×10^13}, {7., 7.93007×10^15}, {8., 1.17693×10^18},
 {9., 1.74671×10^20}, {10., 2.59235×10^22}}
```



Out[8]=
```
- Graphics -
```

```
General::ivar : 4 is not a valid variable. More…
```

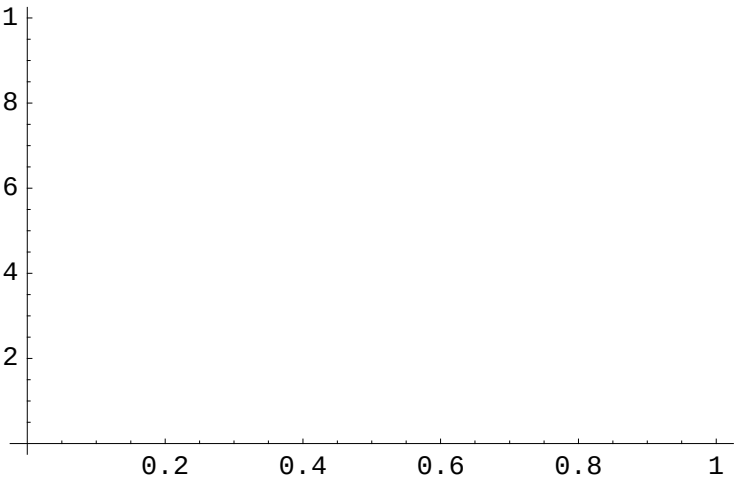
```
ReplaceAll::reps :
 {FindFit[{{0., 10.}, {1., 747.066}, {2., 110137.}, <<5>>, {8.,
       1.17693×10^18}, {9., 1.74671×10^20}, <<1>>}, a + a ℯ^(a x), {a, 4, 5}, x]}
   is neither a list of replacement rules nor a valid dispatch
   table, and so cannot be used for replacing. More…
```

Out[9]=

```
a + a ℯ^(a x) /.
 FindFit[{{0., 10.}, {1., 747.066}, {2., 110137.}, {3., 1.63451×10^7},
   {4., 2.42583×10^9}, {5., 3.60024×10^11}, {6., 5.34324×10^13},
   {7., 7.93007×10^15}, {8., 1.17693×10^18}, {9., 1.74671×10^20},
   {10., 2.59235×10^22}}, a + a ℯ^(a x), {a, 4, 5}, x]
```

```
General::ivar : 4 is not a valid variable. More…

ReplaceAll::reps :
 {FindFit[{{0., 10.}, {1., 747.066}, {2., 110137.}, <<5>>, {8.,
       1.17693×10^18}, {9., 1.74671×10^20}, <<1>>}, a + <<1>>, <<1>>,
     4.16667×10^-7]} is neither a list of replacement rules nor a
   valid dispatch table, and so cannot be used for replacing. More…

General::ivar : 4.` is not a valid variable. More…

ReplaceAll::reps :
 {FindFit[{{0., 10.}, {1., 747.066}, {2., 110137.}, <<5>>, {8.,
       1.17693×10^18}, {9., 1.74671×10^20}, <<1>>}, a + <<1>>, <<1>>,
     4.16667×10^-7]} is neither a list of replacement rules nor a
   valid dispatch table, and so cannot be used for replacing. More…

General::ivar : 4 is not a valid variable. More…

General::stop : Further output of
   General::ivar will be suppressed during this calculation. More…

ReplaceAll::reps :
 {FindFit[{{0., 10.}, {1., 747.066}, {2., 110137.}, <<5>>, {8.,
       1.17693×10^18}, {9., 1.74671×10^20}, <<1>>}, a + <<1>>, <<1>>,
     4.16667×10^-7]} is neither a list of replacement rules nor a
   valid dispatch table, and so cannot be used for replacing. More…

General::stop : Further output of ReplaceAll::reps will
   be suppressed during this calculation. More…

Plot::plnr : fitfunc2[x] is not a machine-
   size real number at x = 4.1666666666666667`*^-7. More…

Plot::plnr : fitfunc2[x] is not a machine-
   size real number at x = 0.40566991572915795`. More…

Plot::plnr : fitfunc2[x] is not a machine-
   size real number at x = 0.8480879985937368`. More…
```

```
General::stop : Further output of
   Plot::plnr will be suppressed during this calculation. More…
```



Out[10]= - Graphics -

Again, it is quite obvious that the fitting equation does not work.

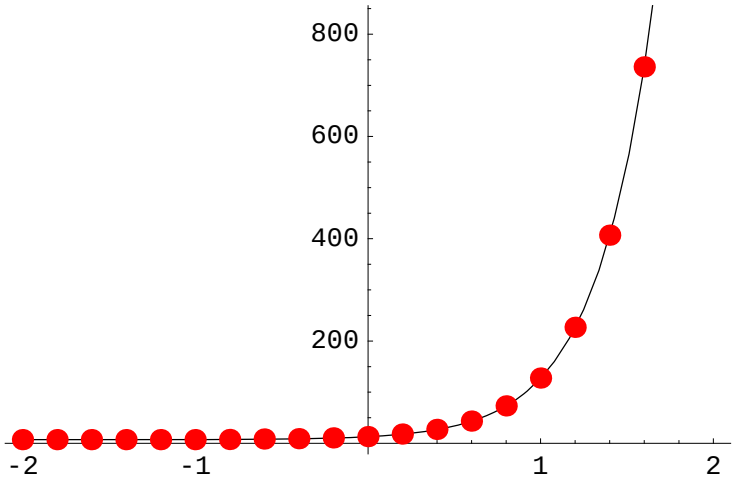
■ **The examples below are more advanced and are focused on fitting and not importing. They are provided by the technical staff (Harry Calkin) at Wolfram's. Harry has provided several approaches, the first of which I call Solution 1:**

I've attached the email exchange below so you have some idea of what is going on.

In[23]:=
```
Clear[func, fitfunc]

func = Table[{x, 7 + 6 Exp[3 x]}, {x, -2, 2, .2}]
fitfunc = a + b Exp[c x] /. FindFit[func, a + b Exp[c x], {a, b, c}, x]

plotfit = Plot[fitfunc, {x, -2, 2}, Epilog -> {
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ func}]
```

Out[24]=
$$\left\{\left\{-2, 7 + \frac{6}{e^6}\right\}, \{-1.8, 7.0271\}, \{-1.6, 7.04938\}, \{-1.4, 7.08997\}, \{-1.2, 7.16394\}, \{-1., 7.29872\}, \{-0.8, 7.54431\}, \{-0.6, 7.99179\}, \{-0.4, 8.80717\}, \{-0.2, 10.2929\}, \{0., 13.\}, \{0.2, 17.9327\}, \{0.4, 26.9207\}, \{0.6, 43.2979\}, \{0.8, 73.1391\}, \{1., 127.513\}, \{1.2, 226.589\}, \{1.4, 407.118\}, \{1.6, 736.063\}, \{1.8, 1335.44\}, \{2., 2427.57\}\right\}$$

Out[25]=
$$7. + 6. e^{3. x}$$

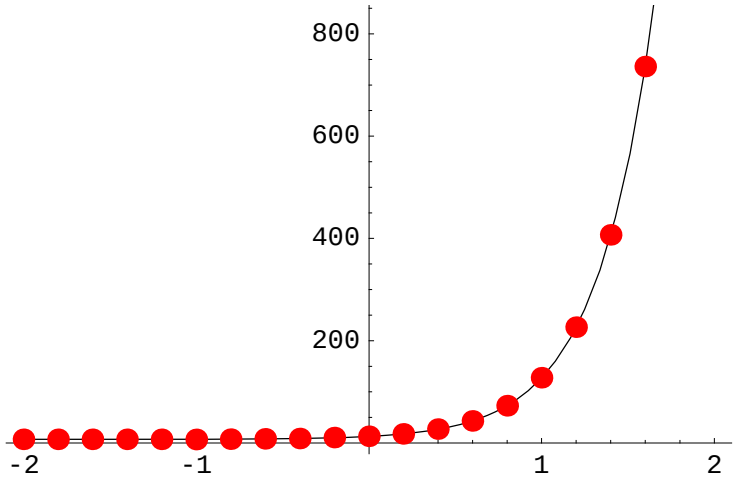


Out[26]=
```
- Graphics -
```

■ Harry Calkin's Solution 2:

In[107]:=
```
<< Graphics`
func = Table[{x, 7 + 6 Exp[3 x]}, {x, -2, 2, .2}];
fitfunc = a + b Exp[c x] /. FindFit[func, a + b Exp[c x], {a, b, c}, x]
DisplayTogether[{Plot[fitfunc, {x, -2, 2}], ListPlot[func, PlotStyle ->
{RGBColor[1, 0, 0], PointSize[.03]}]}]
```

Out[109]=
$$7. + 6. e^{3. x}$$



Out[110]=
```
- Graphics -
```

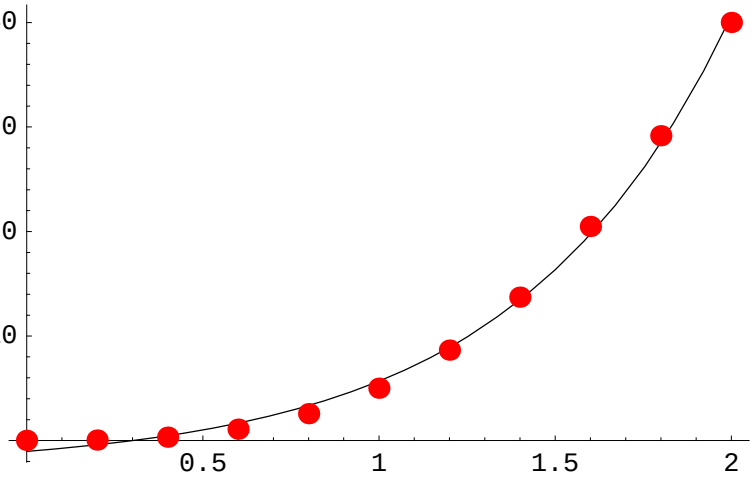
■ Using Wolfram's (Harry Calkin's) Solution 1:

Fitting a Power Law to an exponential: Part 1

With the variable a.

In[111]:=
```
<< Graphics`
Clear[func, fitfunc]
n = 3;
y[x_] = 5 x^n;
func = Table[{x, y[x]}, {x, 0, 2, .2}];

fitfunc = a + b Exp[c x] /. FindFit[func, a + b Exp[c x], {a, b, c}, x]

plotfit = Plot[fitfunc, {x, 0, 2}, Epilog -> {
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ func}]
(*Epilog puts stuff on the plot after it has been plotted. In this case,
 it puts red points for the func function*)
```

Out[116]=
$$-2.62607 + 1.59784\, e^{1.64934\, x}$$



Out[117]=
```
- Graphics -
```

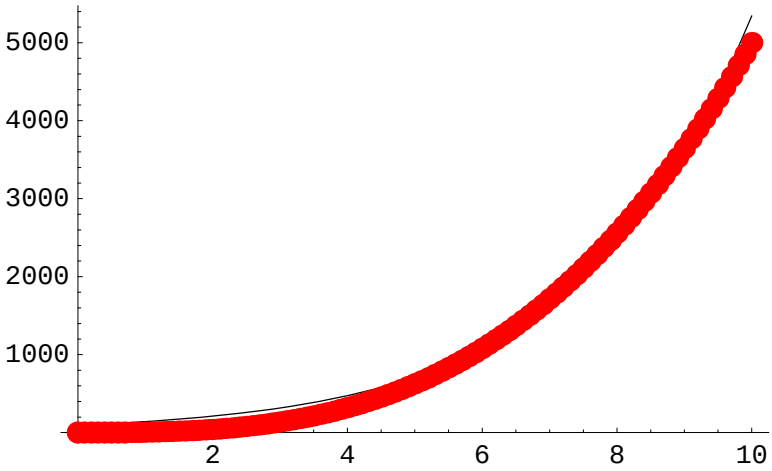
■ Using Wolfram's (Harry Calkin's) Solution 1:

Fitting a Power Law to an exponential: Part 2

Without the variable a.

In[7]:=
```
<< Graphics`
Clear[func, fitfunc]

n = 3;
func = Table[{x,  5 x^n}, {x, 0, 10, .100}];

fitfunc = b Exp[c x] /. FindFit[func, b Exp[c x], {b, c}, x]

plotfit = Plot[fitfunc, {x, 0, 10}, Epilog -> {
  RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ func}]
```

Out[11]=
$$94.3383\, e^{0.403634\, x}$$



Out[12]=
```
- Graphics -
```

Using Wolfram's (Harry Calkin's) Solution 1:

■ Fitting an exponential to a Power Law.

In this example, "start values" are given to obtain a fit.

Hi Harry,

You solutions work very well for all my original examples except one. I can use your approach fitting a power function (ax^n) with an exponential. However, I run out of luck trying to fit an exponential to a power function (see last few cells in the attached file). Any help would be great.

Thank you,

Bill

Hello,

For examples of this sort you will often have to give start values for the parameters as the resulting merit function may have several local minima. The following gives good results for your example. The two starting values for each parameter causes the program to use a secant method in place of a gradient to find the minimum for the merit function.

```
fitfunc[x_] = a + b*x^n /. FindFit[func, a + b*x^n, {{a, 3, 5}, {b, 1, 4},
{n, 5, 8}}, x]
```

You can get the residual sum of squares for this fit using the following.

```
Map[(fitfunc[#[[1]]] - #[[2]])^2 &, func] // Total
```

Sincerely,

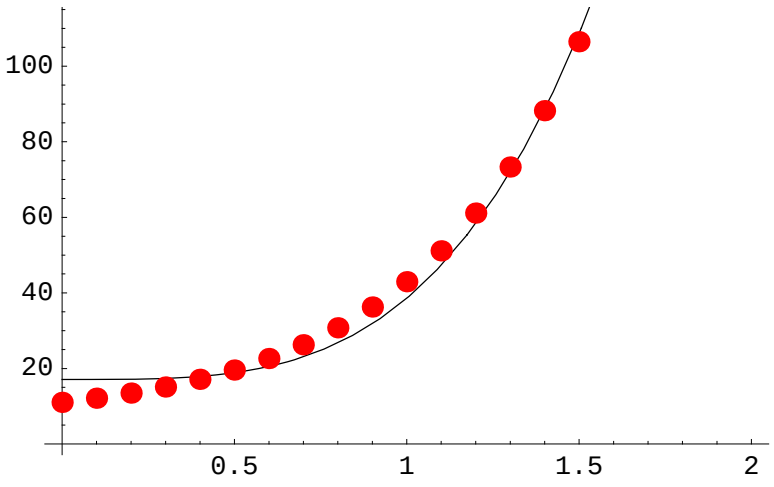
harry Calkins

Technical Support

Wolfram Research, Inc

In[1]:=

```
<< Graphics`
Clear[func, fitfunc, n, a, b, c, x, plotfit]
func = Table[{x, 6 + 5 Exp[2 x]}, {x, 0, 2, .1}];

(* Fitting Function *)
fitfunc[x_] =
  a + b * x^n /. FindFit[func, a + b * x^n, {{a, 3, 5}, {b, 1, 4}, {n, 5, 8}}, x];

(* You can get the residual sum of squares for this fit using
  the following *)
residuals = Map[(fitfunc[#[[1]]] - #[[2]])^2 &, func] // Total;

plotfit = Plot[fitfunc[x], {x, 0, 2},
   Epilog -> {RGBColor[1, 0, 0], AbsolutePointSize[8], Point /@ func}];

Print["Data = ", MatrixForm[func]]
Print["Best Fit = ", fitfunc[x]]
Print["Residual of Sum of Squares for Fit = ", residuals]
```



$$\text{Data} = \begin{pmatrix} 0 & 11 \\ 0.1 & 12.107 \\ 0.2 & 13.4591 \\ 0.3 & 15.1106 \\ 0.4 & 17.1277 \\ 0.5 & 19.5914 \\ 0.6 & 22.6006 \\ 0.7 & 26.276 \\ 0.8 & 30.7652 \\ 0.9 & 36.2482 \\ 1. & 42.9453 \\ 1.1 & 51.1251 \\ 1.2 & 61.1159 \\ 1.3 & 73.3187 \\ 1.4 & 88.2232 \\ 1.5 & 106.428 \\ 1.6 & 128.663 \\ 1.7 & 155.821 \\ 1.8 & 188.991 \\ 1.9 & 229.506 \\ 2. & 278.991 \end{pmatrix}$$

$$\text{Best Fit} = 17.0763 + 21.494\, x^{3.5736}$$

$$\text{Residual of Sum of Squares for Fit} = 263.321$$

Hi Harry,

You solutions work very well for all my original examples except one. I can use your approach fitting a power function (ax^n) with an exponential. However, I run out of luck trying to fit an exponential to a power function (see last few cells in the attached file). Any help would be great.

Thank you,

Bill

Hello,

For examples of this sort you will often have to give start values for the parameters as the resulting merit function may have several local minima. The following gives good results for your example. The two starting values for each parameter causes the program to use a secant method in place of a gradient to find the minimum for the merit function.

fitfunc[x_] = a + b*x^n /. FindFit[func, a + b*x^n, {{a, 3, 5}, {b, 1, 4},
{n, 5, 8}}, x]

You can get the residual sum of squares for this fit using the following.

Map[(fitfunc[#[[1]]] - #[[2]])^2 &, func] // Total

Sincerely,

harry Calkins
Technical Support
Wolfram Research, Inc

In[133]:=
```
fitfunc[x_] =
 a + b * x^n /. FindFit[func, a + b * x^n, {{a, 3, 5}, {b, 1, 4}, {n, 5, 8}}, x]

(*You can get the residual sum of squares for this fit using
  the following.*)
Map[(fitfunc[#[[1]]] - #[[2]])^2 &, func] // Total
```

Out[133]=
$$16.9281 + 21.2078\, x^{3.60645}$$

Out[134]=
```
155.434
```

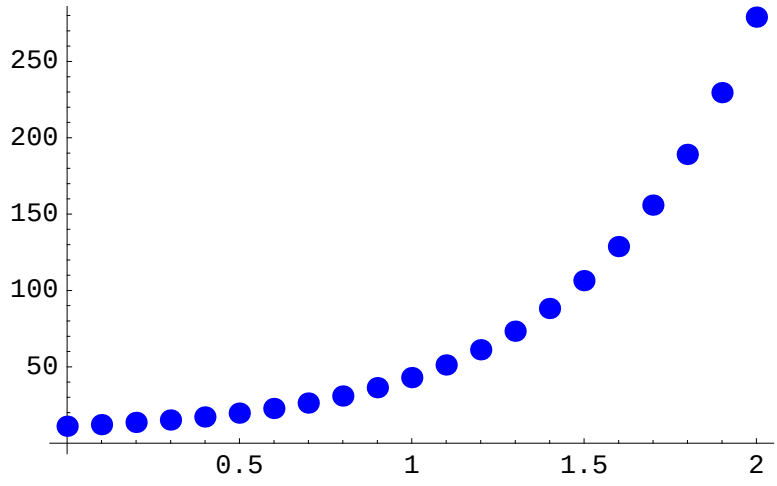
■ **If you just have a table of data, then take the following approach.**

In[10]:=

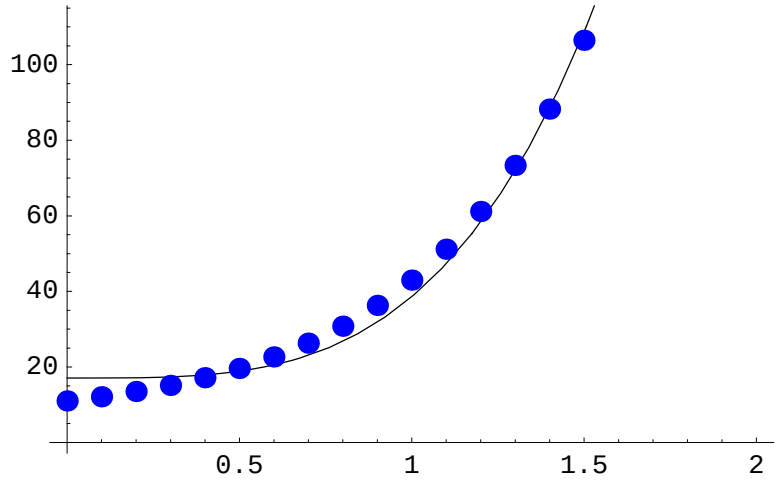
```
Clear[table, fittable, DetermineFit, plotfit, a, b, x, n]
```

```
table = {{0, 11},
 {0.1`, 12.107013790800849`},
 {0.2`, 13.459123488206352`},
 {0.30000000000000004`, 15.110594001952546`},
 {0.4`, 17.12770464246234`},
 {0.5`, 19.591409142295227`},
 {0.6000000000000001`, 22.60058461368274`},
 {0.7000000000000001`, 26.275999834223377`},
 {0.8`, 30.765162121975578`},
 {0.9`, 36.248237322064725`},
 {1.`, 42.945280494653254`},
 {1.1`, 51.12506749717061`},
 {1.2000000000000002`, 61.115881903208034`},
 {1.3`, 73.31869017500846`},
 {1.4000000000000001`, 88.22323385548526`},
 {1.5`, 106.42768461593836`},
 {1.6`, 128.66265098554678`},
 {1.7000000000000002`, 155.82050023698514`},
 {1.8`, 188.99117221838992`},
 {1.9000000000000001`, 229.50592246650422`},
 {2.`, 278.9907501657212`}};
```

```
ListPlot[table, PlotStyle -> {RGBColor[0, 0, 1], AbsolutePointSize[8]}]
(* Fitting Function *)
fittable[x_] =
  a + b * x^n /. FindFit[table, a + b * x^n, {{a, 3, 5}, {b, 1, 4}, {n, 5, 8}},
    x];
plotfit = Plot[fittable[x], {x, 0, 2},
   Epilog -> {RGBColor[0, 0, 1], AbsolutePointSize[8], Point /@ table}];
(* You can get the residual sum of squares for this fit using
  the following *)
residuals = Map[(fittable[#[[1]]] - #[[2]])^2 &, func] // Total;

Print["Data = ", MatrixForm[table]]
Print["Best Fit = ", fittable[x]]
Print["Residual of Sum of Squares for Fit = ", residuals]
```



Out[12]= - Graphics -



Total::normal :
Nonatomic expression expected at position 1 in Total[func]. More…

$$\text{Data} = \begin{pmatrix} 0 & 11 \\ 0.1 & 12.107 \\ 0.2 & 13.4591 \\ 0.3 & 15.1106 \\ 0.4 & 17.1277 \\ 0.5 & 19.5914 \\ 0.6 & 22.6006 \\ 0.7 & 26.276 \\ 0.8 & 30.7652 \\ 0.9 & 36.2482 \\ 1. & 42.9453 \\ 1.1 & 51.1251 \\ 1.2 & 61.1159 \\ 1.3 & 73.3187 \\ 1.4 & 88.2232 \\ 1.5 & 106.428 \\ 1.6 & 128.663 \\ 1.7 & 155.821 \\ 1.8 & 188.991 \\ 1.9 & 229.506 \\ 2. & 278.991 \end{pmatrix}$$

$$\text{Best Fit} = 17.0763 + 21.494\, x^{3.5736}$$

$$\text{Residual of Sum of Squares for Fit} = \text{Total}[\text{func}]$$