Extrinsic S/C Prop.: Free Carrier Statistics

- Free Carrier Statistics:
  - ✓ Previously, we determined the eqns:
    - o $n = f(T)$
    - o $p = f(T)$
  - ✓ Carrier concentrations as a function of temperature in intrinsic S/C's.
  - ✓ We found that:

$$n_i^2 = N_c N_v e^{-E_g/kT}$$

  - ✓ Since we have been discussing impurities in S/Cs (extrinsic S/Cs), we would like to find *n* and *p* for the extrinsic case.
  - ✓ That is, we want to determine how many free e^-'s and h^+'s exist near their respective band edges a a function of:
    - o Impurity concentration
    - o Temperature
  - ✓ Called "Free Carrier Statistics", this is again performed using:
    - o Statistical Mechanics
    - o Quantum Mechanics

Extrinsic S/C Prop.: Free Carrier Statistics

- Let's define our extrinsic S/C system:
  - ✓ One type of donor (when I say "donor" or "acceptor", I mean shallow-level donor or acceptor):
    - o The donor has energy, E_D
    - o The donor concentration, N_D
  - ✓ One type of acceptor
    - o The acceptor has energy, E_A
    - o The acceptor concentration, N_A
  - ✓ The position of the Fermi energy level, E_f, will be determined self-consistently by the:
    - o *Temperature, T*
    - o *Impurity concentrations,* N_D & N_A
    - o *Band gap energy,* E_g
  - ✓ One additional factor to introduce: Degeneracy, *g*, in *k*-space:
    - o Donors:
      - $g = 2$, (spin ½) $m_j = \pm½$ (2 spin states)
    - o Acceptors:
      - $g = 4$, (spin 3/2) $m_j = \pm 3/2$ (4 spin states)
        - » m_j is the total angular momentum

Extrinsic S/C Prop.: Free Carrier Statistics

- We now can ask: "What is the probability that a donor level at energy E_D with degeneracy g is occupied?"
- In order to answer this question, we need to use a distribution statistics of e^-'s & h^+'s
- What distribution function would you suggest?

Extrinsic S/C Prop.: Free Carrier Statistics

- The Fermi-Dirac distribution function is (w/degeneracy g):

$$F(E) = \frac{1}{1 + g e^{(E-E_f)/kT}}$$

- The probability for occupancy of a donor level at energy E_D is:

$$F(E_D) = \frac{1}{1 + g_D e^{(E_D-E_f)/kT}}$$

- The fraction of donor levels, N^*, which are occupied by e^-s is:

$$N^* = N_D \cdot F(E_D) = \frac{N_D}{1 + g_D e^{(E_D-E_f)/kT}}$$

- Let's choose a specific case in which:

$$N_D > N_A$$

- What "type" of extrinsic S/C do we have?

Extrinsic S/C Prop.: Free Carrier Statistics

- The maximum number of e⁻'s that can be promoted to the CB is:

$$N = N_D - N_A$$

- There are N_A e⁻'s compensated by acceptors.
- Since the acceptor states lie very close to the VB, the compensated e⁻'s in the acceptor states need nearly the E_g to be promoted to the CB.
- Therefore, they can be safely neglected at low temperatures: $T <$ room temperature
- Let: n = number or concentration of e⁻'s in CB
- Thus: $N = N_D - N_A = N^* + n$
  - ✓ Where N^* is concentration of donor states occupied by e⁻'s

MSE 310-ECE 340
Elec Props of Matls

Extrinsic S/C Prop.: Free Carrier Statistics

- Since: $N^* = N_D \cdot F(E_D)$
- Substituting N^* in the previous equation:

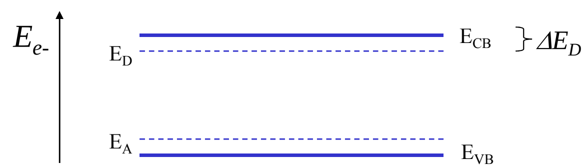
$$N = \frac{N_D}{1 + g_D e^{(E_D - E_f)/kT}} + n$$

$$= \frac{N_D}{1 + g_D e^{E_D/kT} e^{-E_f/kT}} + n$$

- Thus:

$$\frac{N - n}{N_D} = \frac{1}{1 + g_D e^{E_D/kT} e^{-E_f/kT}}$$

- Let's draw the energy band diagram when the S/C is compensated and define certain energy levels.



Extrinsic S/C Prop.: Free Carrier Statistics

- Continuing with our previous equation and taking the reciprocal of both sides, we have:

$$\frac{N_D}{N-n} = 1 + g_D e^{E_D/kT} e^{-E_f/kT}$$

$$\frac{N_D}{N-n} - 1 = g_D e^{E_D/kT} e^{-E_f/kT}$$

$$\frac{N_D - N + n}{N-n} = g_D e^{E_D/kT} e^{-E_f/kT} \qquad \text{(i)}$$

- From our earlier determination of the number of e^-'s in the CB, we found that:

$$n = N_C e^{-(E_c - E_f)/kT}$$

- Where we assumed the *Boltzmann Approximation*.
- Thus, we are assuming that the S/C is nondegenerate.

Extrinsic S/C Prop.: Free Carrier Statistics

- Solving for E_f, we have:

$$E_f = E_c + kT \ln\left(\frac{n}{N_C}\right)$$

- Substituting E_f into equation. (i), we obtain:

$$\frac{N_D - N + n}{N-n} = g_D e^{-(E_C - E_D)/kT} \frac{\mathrm{N_C}}{\mathrm{n}}$$

$$Let: \ \Delta E_D = E_C - E_D \ \& \ \Theta = \mathrm{N_C} g_D e^{-\Delta E_D/kT}$$

$$So: \ \frac{n(N_D - N + n)}{N-n} = \Theta \quad \text{(ii)}$$

- With some algebra, we obtain a quadratic equation in *n*:

$$n^2 + (N_D + \Theta - N)n - N\Theta = 0$$

Extrinsic S/C Prop.: Free Carrier Statistics

- To determine the number of e^-'s in CB, solve for *n*.
- Using the Quadratic equation, solve for *n*:

$$n = \frac{1}{2}\left\{-\left(N_D + \Theta - N\right) \pm \sqrt{\left(N_D + \Theta - N\right)^2 + 4\Theta N}\right\}$$

$$= \frac{1}{2}\left\{-\left(N_D + \Theta - N\right) \pm \left(N_D + \Theta - N\right)\sqrt{1 + \frac{4\Theta N}{\left(N_D + \Theta - N\right)^2}}\right\}$$

$$= \frac{1}{2}\left(N_D + \Theta - N\right)\left\{\pm\sqrt{1 + \frac{4\Theta N}{\left(N_D + \Theta - N\right)^2}} - 1\right\}$$

- Since $n \geq 0$, we take only the positive solution.

$$n = \frac{1}{2}\left(N_D + \Theta - N\right)\left\{\sqrt{1 + \frac{4\Theta N}{\left(N_D + \Theta - N\right)^2}} - 1\right\}$$

$$Where: \quad \Theta = N_C g_D e^{-\Delta E_D / kT}$$

- This last expression is good to within several *kT* of the CB b/c the *Boltzmann Approximation* was used.

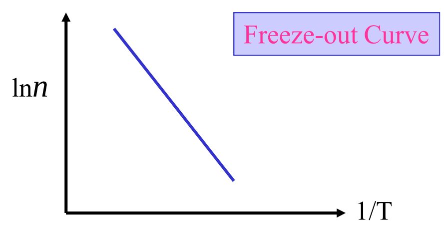
Extrinsic S/C Prop.: Free Carrier Statistics

We can use equation (ii) to examine several *Cases*:

- Case 1: $N_A = 0$ therefore $N = N_D$.
  - ✓ @ low T, $n << N_D$.
  - ✓ n = concentration of CB e^-'s, a.k.a. the free carrier concentration

$$n \approx \sqrt{g_D N_D N_C e^{-\Delta E_D / kT}} = \left(g_D N_D N_C\right)^{1/2} e^{-\Delta E_D / 2kT}$$

✓Plot of ln(n) vs 1/T



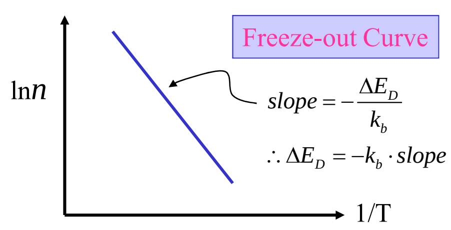
MSE 310-ECE 340
Elec Props of Matls

Extrinsic S/C Prop.: Free Carrier Statistics

- Case 2: $N_A \neq 0$. T~low, thus $n << N_A < N_D$

$$n \approx g_D N_C \frac{N_D - N_A}{N_A} e^{-\Delta E_D / kT}$$

✓ Plot of ln(n) vs 1/T



✓ This region is also known as the "*Full-slope regime*".

MSE 310-ECE 340
Elec Props of Matls

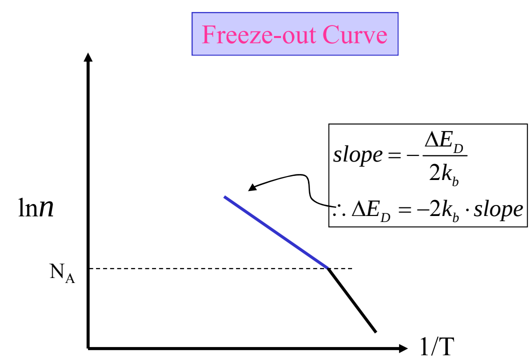
Extrinsic S/C Prop.: Free Carrier Statistics

- Case 3: Increase T such that $N_A << n < N_D$

$$n \approx (g_D N_C N_D)^{1/2} e^{-\Delta E_D / 2kT}$$

✓ Similar to case 1, we find that the slope is $E_D/2k$ or "half-slope".

✓ Hence, the region in blue below is known as the "*half-slope regime*".

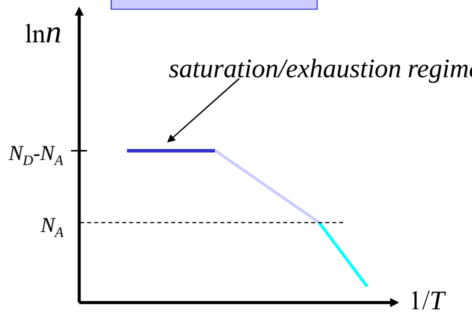


Extrinsic S/C Prop.: Free Carrier Statistics

- Case 4:
  - As T continues to increase, a region is attained in which the free e^- concentration, n, remains constant.
  - This is known as the
    - Exhaustion regime
    - Saturation regime
  - In this condition, n is given by:

$$n = N_D - N_A$$

Freeze-out Curve



  - Question: What happens at higher temps beyond the saturation regime?

Extrinsic S/C Prop.: Free Carrier Statistics

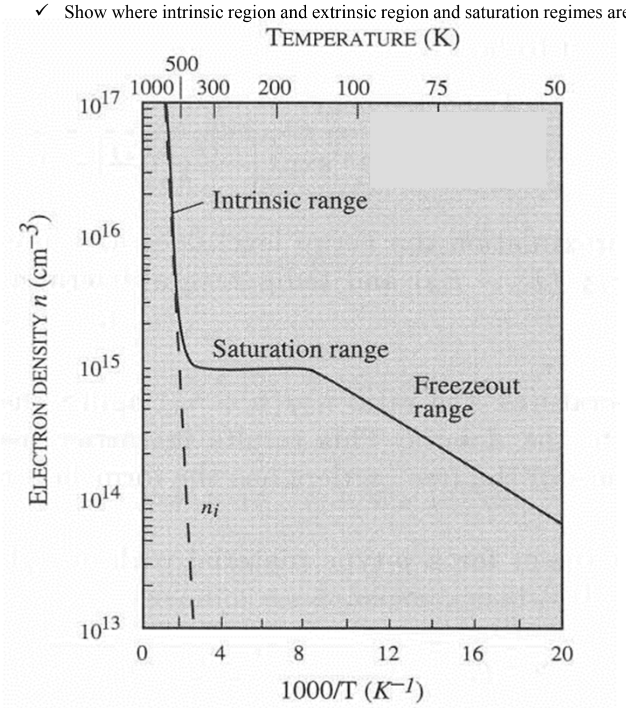
- Fermi Energy Level of a Doped S/C as a Function of Temperature
  - Assume equilibrium
  - At temperatures sufficiently high for the S/C to be intrinsic, E_f lies near mid-gap.
  - As T decreases E_f moves either toward the:
    - CB for n-type
    - VB for p-type
  - E_f coincides with the majority dopant energy level at the T for which 50% of the majority dopants are frozen out (i.e., neutral).

$$\therefore N_D = N_D^o - N_D^+$$
$$N_D^o = N_D^+$$

  - For *uncompensated* S/C (ideal case), E_f would approach a position close to the mid-point between the dopant level and a band edge (CB or VB).
  - For compensated S/Cs, a fraction of the majority dopants remain ionized at the lowest T and E_f will be very close to the majority dopant energy level.

Extrinsic S/C Prop.: Free Carrier Statistics

- In-class Exercise:
  - ✓ Determine if n- or p-type?
  - ✓ $|N_D-N_A|$?
  - ✓ Minority dopant concentration?
  - ✓ Full slope or half slope?
  - ✓ E_g? What is the S/C material?
  - ✓ E_D or E_A? What is the dopant?
  - ✓ Show where intrinsic region and extrinsic region and saturation regimes are.



Extrinsic S/C Prop.: Free Carrier Statistics

- Freezeout Curve

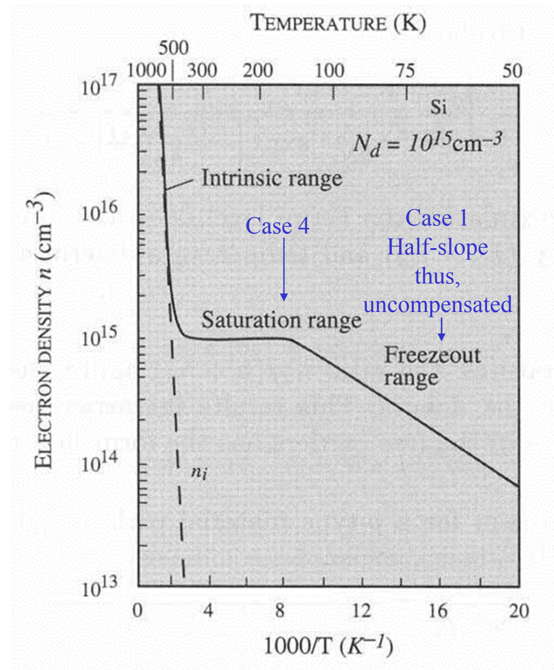


Figure 2.15: Electron density as a function of temperature for a Si sample with donor impurity concentration of 10^{15} cm^{-3}. It is preferable to operate devices in the saturation region where the free carrier density is approximately equal to the dopant density.

Singh, *Semiconductor Devices: Basic Principles* (2000)

Extrinsic S/C Prop.: Free Carrier Statistics

- Freezeout Curve

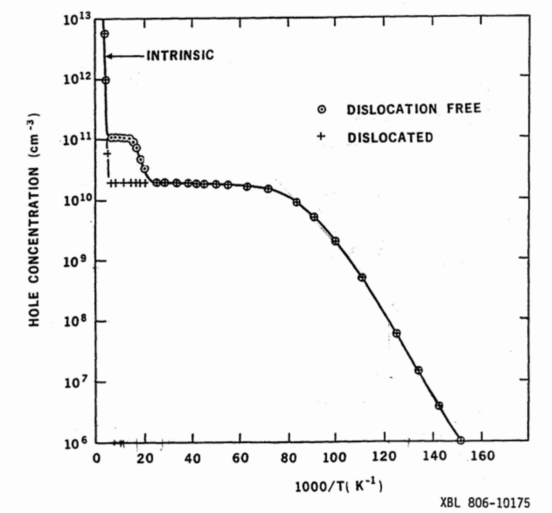


Fig. 21. Arrhenius plots of the free hole concentrations of a dislocation-free and a dislocated piece of single crystal high-purity Ge grown in a H_2 atmosphere.

McCluskey & Haller, *Dopants & Defects in Semiconductors* (CRC Press, 2012) fig. 4.13, p. 120
After: Haller, Hansen, & Goulding, **Adv. Phys**. (1981)