TABLE 5.1 MAXWELL-BOLTZMANN INTEGRALS

The second column below gives the value of the integral in the first column. In the third column are listed values of the integral when α has the value $m/2kT$, as is usual in the velocity distributions. The fourth column gives the result in the third column expressed as a multiple of the mean velocity,

$$\bar{v} = \sqrt{8kT / \pi m}.$$

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \qquad = \frac{1}{2}\sqrt{\frac{2\pi kT}{m}} \qquad = \frac{\pi \bar{v}}{4}$$

$$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha} \qquad = \frac{kT}{m} \qquad = \frac{\pi \bar{v}^2}{8}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}} \qquad = \frac{\sqrt{\pi}}{4}\left(\frac{2kT}{m}\right)^{3/2} \qquad = \frac{\pi^2 \bar{v}^3}{32}$$

$$\int_0^\infty x^3 e^{-\alpha x^2} dx = \frac{1}{2\alpha^2} \qquad = \frac{1}{2}\left(\frac{2kT}{m}\right)^{2} \qquad = \frac{\pi^2 \bar{v}^4}{32}$$

$$\int_0^\infty x^4 e^{-\alpha x^2} dx = \frac{3}{8\alpha^2}\sqrt{\frac{\pi}{\alpha}} \qquad = \frac{3\sqrt{\pi}}{8}\left(\frac{2kT}{m}\right)^{5/2} \qquad = \frac{3\pi^3 \bar{v}^5}{256}$$

$$\int_0^\infty x^5 e^{-\alpha x^2} dx = \frac{1}{\alpha^3} \qquad = \left(\frac{2kT}{m}\right)^{3} \qquad = \frac{\pi^3 \bar{v}^6}{64}$$

$$\int_0^\infty x^6 e^{-\alpha x^2} dx = \frac{15}{16\alpha^3}\sqrt{\frac{\pi}{\alpha}} \qquad = \frac{15\sqrt{\pi}}{16}\left(\frac{2kT}{m}\right)^{7/2} \qquad = \frac{15\pi^4 \bar{v}^7}{2^{11}}$$

$$\int_0^\infty x^7 e^{-\alpha x^2} dx = \frac{3}{\alpha^4} \qquad = 3\left(\frac{2kT}{m}\right)^{4} \qquad = \frac{3\pi^4 \bar{v}^8}{256}$$

$$\int_0^\infty x^8 e^{-\alpha x^2} dx = \frac{105}{32\alpha^4}\sqrt{\frac{\pi}{\alpha}} \qquad = \frac{105\sqrt{\pi}}{32}\left(\frac{2kT}{m}\right)^{9/2} \qquad = \frac{105\pi^5 \bar{v}^9}{2^{14}}$$