P.S. #6

$$\int d^3x_1\, d^3x_2\, r_1^m r_2^n \frac{1}{|X_1 - X_2|} e^{-\alpha(r_1+r_2)} = (4\pi)^2 \alpha^{-m-n-5} C_{mn}$$

$C_{mn} = \frac{5}{4}$ $m=n=0$

$= \frac{25}{8}$ $m=0\ n=1$

$= \frac{33}{4}$ $m=n=1$

Introduction to quantum mechanics

B. H. Bransden and C. J. Joachain

$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$\int_0^\infty e^{-ax^2} dx = \left(\frac{\pi}{4a}\right)^{\frac{1}{2}}$

$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1\cdot 3\cdot 5 \cdots (2n-1)}{2^{n+1} a^n}\left(\frac{\pi}{a}\right)^{\frac{1}{2}}$

$\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$

$\int_{-\infty}^\infty dx\, e^{-x^2/a^2} = a\sqrt{\pi} = \sqrt{\pi a^2}$

$\int_{-\infty}^\infty x^2 e^{-x^2/a^2} = \frac{1}{2}a^3\sqrt{\pi} = \frac{1}{2}(a^2)^{3/2}\sqrt{\pi}$

$\int_{-\infty}^\infty x^4 e^{-x^2/a^2} = \frac{3}{4}a^5\sqrt{\pi} = \frac{3}{4}(a^2)^{5/2}\sqrt{\pi}$

$\int_{-\infty}^\infty x^n e^{-x^2/a^2} = 0$ $n = \text{ODD}$

$\int_{-\infty}^\infty x^n e^{-x^2/a} dx = \frac{n-1}{n} a^{\frac{n+1}{2}}\sqrt{\pi}$ $n = 2, 4$

SEE P. 212 McKenney

$\int_0^a \sin\frac{n\pi x}{a} \sin\frac{m\pi x}{a} dx = \int_0^a \cos\frac{n\pi x}{a}\cos\frac{m\pi x}{a} dx = \frac{a}{2}\delta_{n,m}$

$\int_{-\infty}^\infty e^{-x^2/2a^2} dx = (2\pi a^2)^{\frac{1}{2}}$ $\int_{-\infty}^\infty (-x^2) e^{-ax^2} = -\frac{1}{2}\sqrt{\frac{\pi}{a^3}}$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$\frac{1}{1-x} = 1 + x + x^2 + \cdots$ $x^2 < 1$

$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2!}x^2 \pm \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$ $x^2 < 1$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$

$\sin A \cos B = \frac{1}{2}\sin 2A$

Longman Scientific & Technical

Copublished in the United States with
John Wiley & Sons, Inc., New York

① $\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + C$

② $\int x^n e^{ax} dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1} e^{ax} dx$

SPHERICAL COORD. (FROM CARTESIAN): P. 249

$x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$; $z = r\cos\theta$

$dx\,dy\,dz = dV = r^2\sin\theta\, dr\, d\theta\, d\phi$

$\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \int_0^r r^2 dr = 2\cdot 2\pi \cdot \int_0^r r^2 dr = 4\pi\int_0^r r^2 dr$

$\mathcal{H}$ for H: p. 316