Normalizing WFs

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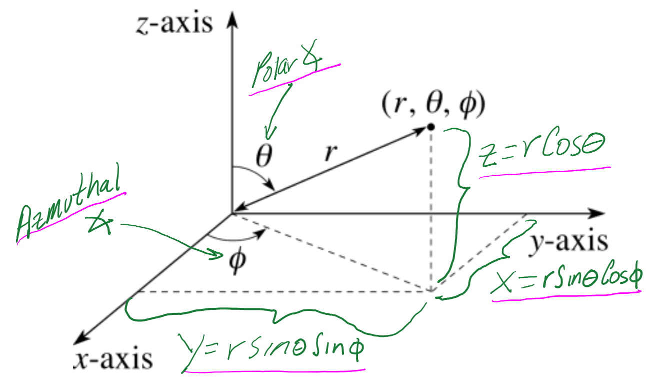
$\psi(r) = e^{-r/a_0}$

Hydrogen atom
r = radial distance
a_0 = Bohr radius



$dV = dl_r\, dl_\theta\, dl_\phi$

$dV = r^2 dr \sin\theta\, d\theta\, d\phi$

$\int dV = \int_0^\infty r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$

Normalyzing a WF

$\langle N\psi | N\psi \rangle = 1$

$\int N^* \psi^* N \psi\, dV = 1$

$|N|^2 \int \psi^* \psi\, dV = 1$

$\psi(r) = e^{-r/a_0}$

$|\psi(r)|^2 = e^{-\frac{2r}{a_0}}$

$|N|^2 \underbrace{\int_0^\infty r^2 e^{-2r/a_0} dr}\ \underbrace{\int_0^\pi \sin\theta\, d\theta}\ \underbrace{\int_0^{2\pi} d\phi}_{(2\pi - 0) = 2\pi} = 1$

Maxwell-Boltzmann Integral

$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

ax

$\frac{2}{a_0}x$ Let $a = \frac{2}{a_0}$

$-[\cos\pi - \cos 0]$

$-[(-1) - 1]$

$-(-2) = 2$

$\int_0^\infty r^2 e^{-ar} dr = \frac{2!}{a^3}$

$= \frac{2}{\left(\frac{2}{a_0}\right)^3}$

$= \frac{a_0^3}{4}$

$|N|^2 \frac{a_0^3}{\cancel{4}} \cdot \cancel{2} \cdot \cancel{2}\pi = 1$

$\pi |N|^2 a_0^3 = 1$ assume $N^* = N$

$\frac{1}{}$

$$N^2 = \frac{1}{\pi a_0^3}$$

$$\boxed{N = \frac{1}{\sqrt{\pi a_0^3}}}$$

$$\boxed{\psi_N(r) = N\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}}$$