Errata Sheet for
An Introduction to Probability Theory and Stochastic Processes
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Chapter 1, page 8, line 8
Change "proportion" to "number".

Chapter 1, page 8, first equation. Change

$$(a_1 + a_2 + a_3)^n = \underbrace{\sum_{\mathbf{k_1=1}}^{n} \sum_{\mathbf{k_2=1}}^{n} \sum_{\mathbf{k_3=1}}^{n}}_{k_1+k_2+k_3=n} \binom{n}{k_1\, k_2\, k_3} a_1^{k_1} \mathbf{a_1}^{k_2} a_3^{k_3}.$$

to

$$(a_1 + a_2 + a_3)^n = \underbrace{\sum_{k_1=0}^{n} \sum_{k_2=0}^{n} \sum_{k_3=0}^{n}}_{k_1+k_2+k_3=n} \binom{n}{k_1\, k_2\, k_3} a_1^{k_1} a_2^{k_2} a_3^{k_3}.$$

Chapter 1, page 8, first equation. Change

$$(a_1 + a_2 + \cdots + a_r)^n = \underbrace{\sum_{\mathbf{k_1=1}}^{n} \sum_{\mathbf{k_2=1}}^{n} \cdots \sum_{\mathbf{k_r=1}}^{n}}_{k_1+k_2+\cdots+k_r=n} \binom{n}{k_1\, k_2\, \cdots\, k_r} a_1^{k_1} a_2^{k_2} \cdots a_r^{k_r}.$$

to

$$(a_1 + a_2 + \cdots + a_r)^n = \underbrace{\sum_{k_1=0}^{n} \sum_{k_2=0}^{n} \cdots \sum_{k_r=0}^{n}}_{k_1+k_2+\cdots+k_r=n} \binom{n}{k_1\, k_2\, \cdots\, k_r} a_1^{k_1} a_2^{k_2} \cdots a_r^{k_r}.$$

Chapter 1, page 10, line 6 from the bottom. Replace "and is 1 is" with "and is 1 if"

Chapter 1, page 11, line 11 from the bottom.
Replace

$$(i_1 \neq i_2 \neq \cdots \neq i_n)$$

with

$$(i_1 \neq i_2 \neq \cdots \neq i_k)$$

Chapter 1, page 19 line 19.
Replace
"... the first k tosses and tails on the next $k - \ell$ tosses."
with

"... the first k tosses." that is, **delete** "and tails on the next $k-\ell$ tosses"

Chapter 1,page 51, last line of Problem 1.

Replace "the set" with "then set"

Chapter 3, page 107, line 20. Replace "$P(A)$ is a good model of A occuring" with "$P(A)$ is a good model for the probability of A occuring".

Chapter 3, page 108, line 12 from the bottom. Replace "with some examples." with "with an example."

Chapter 3, page 126, line 11.

Replace "$A_1, , ..., A_n$" with "$A_1, ..., A_n$".

Restate Theorem 4 as follows:

Theorem 4 $A_1, ..., A_n$ *Independent Their Complements Are Independent*

Let $A_1, A_2, ..., A_n$ be n *independent* events from a probability space $(\Omega, \mathcal{F}, P)$.

Then the n sets $A_1', A_2', ..., A_n'$ are independent where $A_i' = A_i$ or A_i^c.

Chapter 3, page 113. Replace

$$\begin{aligned} P(HS_{1a}) &= P(HS_{1a}|HS_1)P(HS_1) + \underbrace{P(HS_{1a}|HS_1^c)}_{0}P(HS_1^c) \\ &= P(HS_{1a}|HS_1)P(HS_1) \\ &= h_1 p_1. \end{aligned}$$

with

$$\begin{aligned} P(HS_{1a}) &= P(HS_{1a}|HS_1)P(HS_1) \\ &= h_1 p_1. \end{aligned}$$

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Chapter 3, page 129, line 6 from the bottom to page 130 line 23. Replace with the following:

Why are Bernoulli random variables important? Consider the experiment of tossing a biased coin *once*, i.e., $(\Omega_1, \mathcal{F}_1, P_1)$ is our probability space. Suppose we don't know the value of $p > 0$. The number $0 < p < 1$ is the number we associate with the average number (proportion) of heads in n tosses. So to estimate p, *repeat* the experiment n times and *count* the number of heads n_H. Is $p = n_H/n$? To answer this we use the biased coin tossing probability space $(\Omega_n, \mathcal{F}_n, P_n)$ to model *repeated independent trials* of our experiment. That is,

$$\begin{aligned} X_i(\omega) &= 1, \text{ if } H \text{ occurs on the } i^{th} \text{ trial}, \\ X_i(\omega) &= 0, \text{ if } T \text{ occurs on the } i^{th} \text{ trial} \end{aligned}$$

and

$$\begin{aligned} P_n(\{\omega | X_i(\omega) = 1\}) &\triangleq p \\ P_n(\{\omega | X_i(\omega) = 0\}) &\triangleq 1 - p. \end{aligned}$$

Consider the WLLN as given in Problem 4(g) of Chapter 2 (page 97). It states that for arbitrary $\varepsilon > 0$, no matter how small, we have

$$\lim_{n\to\infty} P_n\left(\left\{\omega \in \Omega_n | \ \left|\frac{S_n(\omega)}{n} - p\right| < \epsilon\right\}\right) = 1.$$

What this limit means is that given any $\epsilon > 0$ and any $\delta > 0$, no matter how small, we can find an N large enough such that for *all* $n \geq N$

$$P_n\left(\left\{\omega \in \Omega_n | \ \left|\frac{S_n(\omega)}{n} - p\right| < \epsilon\right\}\right) > 1 - \delta.$$

In words, with n large enough, the number of heads in n tosses, i.e., S_n/n, falls in the range $p \pm \epsilon$ with probability close to 1. Less precisely, with n large enough, this says that the ratio

$$\frac{n_H}{n} = \frac{S_n(\omega)}{n} \approx p$$

except for a set whose probability is less than δ. Thus assigning the probability of heads to be p is consistent with the relative frequency interpretation.

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Chapter 3, page 150, line 4 from the top. Replace "$n > 2$" with "$n \geq 2$"

Chapter 4, page 163, line 12. Replace "of length n" with "of length $1/2^n$"

Chapter 4, page 172, line 16 from the top. Replace

$$F_{U_{o2}}(d)$$

with

$$\lim_{d\to d_i^-} F_{U_{o2}}(d).$$

Chapter 4, page 172, line 3 from the bottom. Replace "an random" with "a random"

Chapter 4, page 179, last line. Replace

$$P(\{\omega|U_{o3}(\omega) = d\}) = P(\{\omega|U_{e3}(\omega) = d\}) = \frac{1}{2^3}.$$

with

$$P(\{\omega|U_{o3}(\omega) = d\}) = P(\{\omega|U_{e3}(\omega) = d\}) = \frac{1}{2^3},$$

That is, replace the period with a comma.

Chapter 4, page 212, last line. Replace "Hint: See problem 19, part (c)" with "Hint: See problem 19, part (d)".

Chapter 5, page 216, lines 5 and 6 from bottom. Replace "$\leq U_o(\omega)$" with "$< U_o(\omega)$"

Chapter 5, page 220, line 2 from bottom. Replace "a unique inverse" with "an inverse"

Chapter 5, page 222, line 1. Replace "...to denote a normal (Gaussian) random variable..." with "...to denote a random variable..."

Chapter 5, page 222, Equation (5.4). Replace "$q < u < 1$" with "$q < u \leq 1$"

Chapter 5, page 223, line 3. Replace "$U_o(\omega) < 1$" with "$U_o(\omega) \leq 1$"
Chapter 5, page 228, line 3 from bottom. Replace "probability as" with "probability distribution as"

Chapter 5, page 230, line 9. Replace "However, if we define a sequence ..." with "Define a sequence ...".

Chapter 5, page 235, line 7 from the bottom. Replace "$P(\{\omega \in 1]|$ " with "$P(\{\omega \in (0,1]|$".

Chapter 5, page 242, last line Replace "$\sum_{k=0}^{\infty} k\frac{\lambda^{k-1}}{k!}e^{-\lambda} = \lambda$" with "$\sum_{k=0}^{\infty} k\frac{\lambda^{k}}{k!}e^{-\lambda} = \lambda$".

Chapter 5, page 243, line 4. Replace "$\sum_{k=0}^{\infty} k^2\frac{\lambda^{k-1}}{k!}e^{-\lambda} = \lambda^2 + \lambda$" with "$\sum_{k=0}^{\infty} k^2\frac{\lambda^{k}}{k!}e^{-\lambda} = \lambda^2 + \lambda$".

Chapter 5, page 243, line 4. Replace "$\sum_{k=0}^{\infty}(k-\lambda)^2\frac{\lambda^{k-1}}{k!}e^{-\lambda} = \lambda$" with "$\sum_{k=0}^{\infty}(k-\lambda)^2\frac{\lambda^{k}}{k!}e^{-\lambda} = \lambda$".

Chapter 6, page 246, line 13 from bottom. Replace "$\sum_{k=0}^{7}$" with "$\sum_{k=1}^{7}$".

Chapter 6, page 250, lines 8 and 9. Missing the right curly bracket "}" in four places.

Chapter 6, page 254, line 6 from the bottom. Replace "$= \int_{-\infty}^{\infty}(x_k - \mu)^2 f_X(x)dx$" with "$= \int_{-\infty}^{\infty}(x - \mu)^2 f_X(x)dx$"

Chapter 6, page 258, line 7 from the bottom. Replace "variables" with "variable"

Chapter 6, page 260, line 6 from the top. Replace "$\mathbb{R}^2 = \bigcup_{i=-\infty}^{\infty} \cdots$" with $\mathbb{R}^2 = \bigcup_{j=-\infty}^{\infty} \cdots$

Chapter 6, page 260, line 11 from the top. Replace "$\Omega = \bigcup_{i=-\infty}^{\infty} \cdots$" with $\Omega = \bigcup_{j=-\infty}^{\infty} \cdots$

Chapter 7, page 277, lines 6,10, and 13. Replace "$T_n(\omega) + T_{n+1}(\omega) > t$" with "$S_n(\omega) + T_{n+1}(\omega) > t$" where

$$S_n(\omega) \triangleq T_1(\omega) + \cdots + T_n(\omega).$$

Chapter 7, page 321, line 12 from the bottom. Replace "$N_k k = 1, 2, 3, ...$" with "$N_k, k = 1, 2, 3, ...$".

Chapter 8, page 325, line 6 from the bottom. Replace "$n < 1$" with "$n \leq 0$"
Chapter 9, page 403, line 11. Replace "define *Erlang*" with "define the *Erlang*".

Chapter 9, page 407, line 8 from the bottome. In the equation replace "$< y$" with "$\leq y$".

Chapter 9, page 409, line 4. In the equation replace "$k < 0$" with "$s < 0$".

Chapter 9, page 410, line 2. In the equation replace "$\frac{1}{\sqrt{na^2/12}}$" with "$\frac{1}{\sqrt{na^2/3}}$".

Chapter 9, page 412, equation (9.52). In the equation replace "s" with "k".

Chapter 9, page 415, Example 21. In the equation replace

$$\text{“}\rho = E[|X_n|^3] = p\text{”}$$

with

$$\text{“}\rho = E[|X_n - \mu_X|^3] = (1-p)^3 p + |-p|^3(1-p)\text{”}.$$

This error propagates through the example. See the complete revision below.

Example 21 *Independent Bernoulli Random Variables*

As before, let X_i be a sequence of independent Bernoulli random variables with parameter $p, 0 < p < 1$. Then

$$\begin{aligned}
\mu_X &= E[X_i] = p, \\
\sigma_X^2 &= var(X_i) = p(1-p), \\
\rho &= E[|X_i - \mu_X|^3] = (1-p)^3 p + |(-p)^3|(1-p) \\
&= p(1-p)((1-p)^2 + p^2),
\end{aligned}$$

and

$$Y_n(\omega) \triangleq \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{X_i(\omega) - \mu_X}{\sigma_X} = \frac{S_n(\omega) - np}{\sqrt{np(1-p)}}.$$

The Berry−Esseen error bound is then

$$M_n \triangleq \sup_{y\in\mathbb{R}} |F_{Y_n}(y) - \Phi(y)| \leq \frac{C\rho}{\sqrt{n}\sigma_X^3} \leq \frac{1}{\sqrt{n}} \frac{0.4784((1-p)^2 + p^2)}{\sqrt{p(1-p)}}.$$

In particular, with $p = 1/2$ we have

$$M_n \triangleq \sup_{y\in\mathbb{R}} |F_{Y_n}(y) - \Phi(y)| \leq \frac{0.4784}{\sqrt{n}}.$$

Thus, if we want $M_n < 0.02$, then we take

$$\frac{0.4784}{\sqrt{n}} \leq 0.02$$

or

$$n \geq \left(\frac{0.4784}{0.01}\right)^2 = 572.17.$$

Thus using $\Phi(y)$ to approximate $F_{Y_n}(y)$ we are assured that the error is no bigger than 0.02 if $n \geq 573$.

Chapter 9, page 416, Example 22, line 8. Replace

$$\text{“} \leq 2\frac{2}{\sqrt{n}}.\text{”}$$

with

$$\text{“} \leq \frac{0.4784}{\sqrt{n}}.\text{”}$$

Chapter 9, page 416, Example 22, lines 13 and 14. Replace
"Taking the error bound into account, we can only say that for $n \geq 10,000$

$$P(\{\omega | p - 0.01 \leq (X_1 + X_2 + \cdots + X_n)/n \leq p + 0.01\}) \geq 0.91$$

as $2\frac{2}{\sqrt{10000}} = 0.04$."

with
"Taking the Berry-Esseen error bound into account, we can still say that for $n \geq 10,000$

$$\text{“}P(\{\omega | p - 0.01 \leq (X_1 + X_2 + \cdots + X_n)/n \leq p + 0.01\}) \geq 0.945$$

as $\frac{0.4784}{\sqrt{10000}} \leq 0.005$."

Chapter 9, page 416, line 3 of Example 23. In the equation replace "$\sigma_U^2 = a^2/12$" with "$\sigma_U^2 = a^2/3$".

Chapter 9, page 416, line 6 of Example 23. In the equation replace "$a^2/12$" with "$a^2/3$".

Chapter 9, page 416, line 6 of Example 23. In the equation replace "$a^2/12$" with "$a^2/3$" and replace "4.972" with "0.6215".

Chapter 9, page 416, Example 23, line 2 from the bottom. Replace "4.972" with "0.6215".

Chapter 9, page 417, Example 24, line 13. Replace

$$\text{“} \leq 2\frac{2}{\sqrt{n}}.\text{”}$$

with

$$\text{“} \leq \frac{0.4784}{\sqrt{n}}.\text{”}$$

Chapter 9, page 418, line 6. Replace

$$\text{“} \leq \frac{4}{\sqrt{n}} + \frac{1}{n}.\text{”}$$

with

$$\text{“} \leq \frac{0.4784}{\sqrt{n}} + \frac{1}{n}.\text{”}$$

Chapter 9, page 424, Problem 9, line 2. Replace "Consider two light bulbs of the same type" with "Consider two different light bulbs of the same type"

Chapter 10, page 432, line 5. Replace "suppose *Jacobian*" with "suppose the *Jacobian*"

Chapter 10, page 450, line 6 from the bottom. Replace "is a small" with "is as small"

Chapter 10, page 454, line 2. Replace "measurement of X to obtain" with "measurement of Y to obtain"

Chapter 10, page 454, line 6 from the bottom. Replace "$\triangleq$" with "$=$"

Chapter 10, page 465, Problem 5, line 7 from the bottom. Replace

$$x_2 = \pm\sqrt{y_1'}$$

with

$$x_2 = \pm\sqrt{y_2'}$$

Chapter 10, page 471, Problem 21, lines 11 and 12. Replace

$$\begin{aligned} f_{XY}(x,y,\rho) &= \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{\left(\left(\frac{x-\mu_X}{\sigma_X}\right)-\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right)^2 - 2(1-\rho)\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)}{2(1-\rho^2)}} \\ &= \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{\left(\left(\frac{x-\mu_X}{\sigma_X}\right)-\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right)^2}{2(1-\rho^2)}} e^{\frac{\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)}{1+\rho}}. \end{aligned}$$

with

$$\begin{aligned} f_{XY}(x,y,\rho) &= \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} e^{-\frac{\left(\left(\frac{x-\mu_X}{\sigma_X}\right)-\rho\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right)^2 + (1-\rho^2)\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2}{2(1-\rho^2)}} \\ &= \frac{1}{\sqrt{2\pi}\sigma_X\sqrt{1-\rho^2}} e^{-\frac{\left(\left(\frac{x-\mu_X}{\sigma_X}\right)-\rho\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right)^2}{2(1-\rho^2)}} \frac{1}{\sqrt{2\pi}\sigma_Y} e^{-\frac{\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2}{2}} \end{aligned}$$

Chapter 10, page 471, Problem 21, line 1 from the bottom. Insert "dz" in the integral.

Chapter 10, page 492, line 14. Replace "number times" with "number of times".

Chapter 11, page 478, First line of equation (11.13). Replace

$$\sigma^2_{X|Y} = E[X|Y = y] = \cdots$$

with

$$\sigma^2_{X|Y} = E[(x - E[X|Y = y])^2] = \cdots$$

Chapter 11, page 502, line 7. Replace "of y, mean square error" with "of y, the mean square error".

Chapter 11, page 503, line 4 from the bottom. Replace "All we know about noise values ..." with "All we know about these noise values ...".

Chapter 11, page 510, line 10 from the bottom. Replace "minimizes both the conditional variance" with "minimizes both"

Chapter 11, page 510, line 7 from the bottom. Replace "and the unconditional variance" with "and"

Chapter 11, page 515, line 11. Replace "$\dfrac{\partial}{\partial a}$" with "$\dfrac{\partial}{\partial \alpha_1}$"

Chapter 11, page 515, line 12. Replace "$\dfrac{\partial}{\partial b}$" with "$\dfrac{\partial}{\partial \alpha_2}$"

Chapter 11, page 517, line 5 in the Remark. Replace "For $i > 1$" with "For $i \geq 1$"

Chapter 11, page 521, line 3. Replace "$\|Y^2\|$" with "$\|Y\|^2$"

Chapter 11, page 532, Problem 1. Replace "$f_{Y|X}(x|y)$" with "$f_{X|Y}(x|y)$"

Chapter 11, page 544, line 3. Replace "it's" with "its".

Chapter 11, page 545, line 12 from the bottom. Replace "$\hat{X}$" with "$\hat{X}^*$".

Chapter 11, page 545, line 10 from the bottom. Replace

$$\text{Hint: Set } E[(X - \hat{X})^2] = E[(X - \hat{X} + \hat{X} - \hat{X})^2]$$

with

$$\text{Hint: Set } E[(X - \hat{X})^2] = E[(X - \hat{X}^* + \hat{X}^* - \hat{X})^2]$$

Chapter 12, page 563, line 8 from the bottom, 2nd equality. Replace "$= \mathbf{x}^T\mathbf{D}\mathbf{y}$" with "$= \mathbf{y}^T\mathbf{D}\mathbf{y}$".

Chapter 12, page 564, line 11. Replace "$= \mathbf{x}^T\mathbf{D}\mathbf{y}$" with "$= \mathbf{y}^T\mathbf{D}\mathbf{y}$".

Chapter 12, page 580, line 9 from the bottom. Replace "$E^*[\mathbf{X}|\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3], , , ,$" with "$E^*[\mathbf{X}|\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3], ...$".

Chapter 13, page 587, line 1. Replace "consists of a" with "consist of an".

Chapter 13, page 590, line 10 from the bottom. Replace "between $j-1$" with "between the $j-1$".

Chapter 13, page 591, line 13. Replace "the n^{th} came" with "the n^{th} arrival came".

Chapter 13, page 613, 2nd line above footnote. Replace "at time n we must" with "at time t we must".

Chapter 13, page 616, last equation. Replace

$$\begin{aligned} F_{V_t}(t') = P(\{\omega|\ V_t(\omega) \le t'\}) &= P(\{\omega|N_t(\omega) - N_{t-t'}(\omega) \ge 1\}) \\ &= \begin{cases} P(\{\omega|N_{t'}(\omega) \ge 1\}), & t \le t', \\ 0, & t > t' \end{cases} \\ &= \begin{cases} 1 - P(\{\omega|N_{t'}(\omega) = 0\}), & t \le t', \\ 0, & t > t' \end{cases} \\ &= \begin{cases} 1 - e^{-\lambda t'}, & t \le t', \\ 0, & t > t' \end{cases} \end{aligned}$$

with

$$\begin{aligned} F_{V_t}(t') = P(\{\omega|\ V_t(\omega) \le t'\}) &= P(\{\omega|N_t(\omega) - N_{t-t'}(\omega) \ge 1\}) \\ &= \begin{cases} P(\{\omega|N_{t'}(\omega) \ge 1\}), & t' \ge 0, \\ 0, & t' < 0 \end{cases} \\ &= \begin{cases} 1 - P(\{\omega|N_{t'}(\omega) = 0\}), & t' \ge 0, \\ 0, & t' < 0 \end{cases} \\ &= \begin{cases} 1 - e^{-\lambda t'}, & t' \ge 0, \\ 0, & t' < 0 \end{cases} \end{aligned}$$

Chapter 13, page 617. Replace equation (13.36) with

$$f_{V_t}(t') = \begin{cases} \lambda e^{-\lambda t'}, & t' \ge 0, \\ 0, & t' < 0. \end{cases} \tag{13.36}$$

Chapter 13, page 627, line 10. Replace "The covariance N_m" with "The covariance of N_m".

Chapter 14, page 653, line 14. Replace

$$\lim_{\delta\to 0^+}(W(t+\delta,\omega)=W(0,\omega)=0.$$

with

$$\lim_{\delta\to 0^+} W(\delta,\omega)=W(0,\omega)=0.$$

Chapter 14, page 655, line 1. Replace "or" with "and".

Chapter 15, page 770, last line. Replace "dt" with "$d\tau$".

Chapter 15, page 771, line 7. Replace

$$\lim_{T\to\infty}\int_{-T}^{T} e^{-j\zeta t}\frac{\sin(\zeta_c\tau)}{\pi\tau}dt$$

with

$$\lim_{T\to\infty}\int_{-T}^{T} e^{-j\zeta t}\frac{\sin(\zeta_c t)}{\pi t}dt$$

Chapter 16, page 789, Figure 16.1. In the lower right-hand plot [in the (2,3) position], the bold line on the abcissa should be solid from 0 all the way to 3/4 instead of stopping between 1/4 and 1/2.

Chapter 16, page 796, Figure 16.4. Remove the "$1/n$" on the $x-$axis".

Chapter 16, page 822, line 6. Replace "$X_k\omega)$" with "$X_k(\omega)$".

Chapter 16, page 823, line 7. End of line. Add a right parenthese ")" inside the last right square bracket.

Chapter 16, page 825, line 9. Replace "in the next subsection" with "in a later subsection".

Chapter 17, page 846, line 4. Replace "dx" with "du".

Chapter 17, page 848, line 5 from the bottom. Replace "... Problem 2 asks the reader is asked to" with "... Problem 2 the reader is asked to".

Chapter 18, page 678, line 1 from the bottom. Replace "$xz+xz$" with "$xy+xz$".

Table of Common Distributions, page 936, row 3, column 1 of the table. Replace "$\frac{\ell q}{p}$" with "$\frac{\ell}{p}$".

Chapter 1, Problem 5

Replace

$$\frac{n+m}{2n} = \frac{1}{2} + \frac{m}{n} \stackrel{n\to\infty}{\to} \frac{1}{2}.$$

with

$$\frac{n+m}{2n} = \frac{1}{2} + \frac{m}{2n} \stackrel{n\to\infty}{\to} \frac{1}{2}.$$

Replace

"... while if m_0 is the largest positive integer such that $\frac{m_o}{n} < \epsilon$ (so $\frac{n+m_0}{2n} = \frac{1}{2} + \frac{m_o}{n} < \frac{1}{2} + \epsilon$)..."

with

"... while if m_0 is the largest positive integer such that $\frac{m_o}{2n} < \epsilon$ (so $\frac{n+m_0}{2n} = \frac{1}{2} + \frac{m_o}{2n} < \frac{1}{2} + \epsilon$)..."