Instructions: Show all steps and logic circuits for full or partial credits. It is very important that you write clearly, so that your test can be graded appropriately and fairly. This is a closed book and closed notes test.

1. (10 points) Show how the function \( f(x_4, x_3, x_2, x_1) = \sum(0,3,5,7,9,13,15) \), where \( x_4 \) is MSB and \( x_1 \) is LSB, can be implemented using

a. one 4-to-16 decoder and an OR gate, and

b. one 4-input LUT.

Name: 

1 of 6
2. (15 points) For the function \( f(x_1, x_2, x_3) = \Sigma m(0, 4, 6, 7) \), where \( x_1 \) is MSB and \( x_3 \) is LSB, use Shannon's expansion to derive an implementation using a 2-to-1 multiplexer and any other necessary gates.

\[
\begin{align*}
\bar{y} &= \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 \\
&= \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 \\
&= \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 \\
&= \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1 + \bar{y}_3 \bar{y}_2 \bar{y}_1
\end{align*}
\]

3. (25 points) Reverse engineer the circuit shown in the schematic in order to derive a two-level realization, where A is MSB and C is LSB.

a. Find the Boolean expression that describes the circuit (Label and find the intermediate expressions, and the final expression, \( Z \)).

\[
\begin{align*}
\bar{g}_1 &= \overline{B \bar{C}} + \bar{B} \overline{C} \\
\bar{g}_2 &= \bar{B} \bar{A} + \overline{A \bar{B}} \\
\bar{g}_3 &= \overline{A \bar{B}} + \bar{A} \overline{B} \\
\bar{g}_4 &= \overline{A \bar{B}} + \bar{A} \overline{B} \\
\bar{g}_5 &= \overline{A \bar{C}} + \overline{A \bar{C}} \\
\bar{g}_6 &= \bar{A} \overline{C} + \overline{A \bar{C}} = \overline{A \bar{C}}
\end{align*}
\]
a. Construct the truth table for the function, Z.

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<th>A</th>
<th>B</th>
<th>C</th>
<th>g_1</th>
<th>g_2</th>
<th>g_3</th>
<th>g_4</th>
<th>g_5</th>
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<th>Z</th>
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b. Write the function in canonical sum-of-products form (the function, Z, from truth table without simplification).

\[ Z = A'B'C + A'BC + AB'C' + ABC \]

c. Simplify the function, Z, using K-maps.

\[ Z = A'C + A'C' \]

= \[ A \oplus C \]
5. (10 points) Show how the function \( f = x_1x_2 + x_1x_3 + x_1x_2x_3 + x_1x_2 + x_1'x_3x_2' \) can be realized using Act 1 logic blocks (Act 1 logic block is shown). Note that there are no NOT gates in the chip; hence complements of signals have to be generated using the multiplexers in the logic block.

\[
\begin{align*}
f &= x_1 (x_3 + x_2 x_3) + x_1 (x_3 + x_2 x_3) + x_1 \left[ x_3 (1 + x_2 + x_2') + x_3 (x_2') \right] \\
&= x_1 \left[ x_3 (x_2) + x_3 (1) \right] + x_1 \left[ x_3 (1 + x_2 + x_2') + x_3 (x_2') \right] \\
&= x_1 \left[ x_3 (x_2) + x_3 (1) \right] + x_1 \left[ x_3 (1) + x_3 (x_2') \right]
\end{align*}
\]
6. (15 points) Complete the following timing diagram.
6. (20 points) Design a shifter circuit, which can shift a four-bit input vector, $W = w_3 w_2 w_1 w_0$, one bit position to the right when the control signals are $S1=0$ and $S0=0$, and one bit position to the left when the control signals are $S1=1$ and $S0=0$. When $S1=0$ and $S0=0$ the output of the circuit should be the same as the input vector. When the condition $S1=1$ and $S0=1$ occurs, the shifter will carry out a right rotate. [Hint: Use four 4-to-1 multiplexers]