Experiment #6: Transformers, Mutual Inductance, and Coupled Coils

1 Objectives

The objectives of this laboratory experiment are:

- To observe the phenomenon of inductive coupling
- To perform polarity and turns ratio tests on a transformer
- To measure the self and mutual inductances of a transformer
- To measure the inductance of two coupled series-aiding or series-opposing coils

2 Theory

Consider the two-winding transformer shown in Figure 1(a) and assume that the primary winding is energized with a current $i_1$ while the secondary winding is open-circuited. The primary current $i_1$ creates a magnetic flux $\phi_1$ in the iron core with two components:

- A large portion of this primary flux, the mutual flux $\phi_{m1}$, is channeled through the magnetic core and links all turns of the secondary winding;
- A small portion of this primary flux, the leakage flux $\phi_{l1}$, links all turns of the primary winding but none of the turns of the secondary winding;

These two fluxes result in the following flux linkages and inductances in each winding:

$$\lambda_1 = N_1\phi_1 = N_1(\phi_{l1} + \phi_{m1}) = L_1i_1 \quad (1)$$
$$\lambda_2 = N_2\phi_{m1} = L_{21}i_1 \quad (2)$$

Similarly, assume that the secondary winding is energized with a current $i_2$ while the primary winding is open-circuited as shown in Figure 1(b). The secondary current $i_2$ creates a magnetic flux $\phi_2$ in the iron core with two components:

- A large portion of this secondary flux, the mutual flux $\phi_{m2}$, is channeled through the magnetic core and links all turns of the primary winding;
- A small portion of this secondary flux, the leakage flux $\phi_{l2}$, links all turns of the secondary winding but none of the turns of the primary winding;

These two fluxes result in the following flux linkages and inductances in each winding:

$$\lambda_1 = N_1\phi_{m2} = L_{12}i_2 \quad (3)$$
$$\lambda_2 = N_2\phi_2 = N_2(\phi_{l2} + \phi_{m2}) = L_{2}i_2 \quad (4)$$
If both windings are energized simultaneously as in Figure 1(c), the resulting flux linkages of each coil are given by:

$$\lambda_1 = N_1 (\phi_{l1} + \phi_{m1} + \phi_{m2}) = L_1 i_1 + L_{12} i_2$$  \hspace{1cm} (5)$$

$$\lambda_2 = N_2 (\phi_{m1} + \phi_{l2} + \phi_{m2}) = L_{21} i_1 + L_2 i_2$$  \hspace{1cm} (6)$$

According to Faraday’s law, the voltages induced in each coil are

$$v_1(t) = R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$  \hspace{1cm} (7)$$

$$v_2(t) = R_2 i_2 + \frac{d\lambda_2}{dt} = R_2 i_2 + L_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$  \hspace{1cm} (8)$$

where $R_1$ and $R_2$ are the resistances of the primary and secondary windings, respectively.
In phasor form, these voltages are given by
\[
\begin{align*}
\tilde{V}_1 &= (R_1 + j\omega L_1)\tilde{I}_1 + j\omega L_{12}\tilde{I}_2 \\
\tilde{V}_2 &= j\omega L_{21}\tilde{I}_1 + (R_2 + j\omega L_2)\tilde{I}_2
\end{align*}
\]
(9) (10)

It will be shown in this experiment that the mutual inductances \(L_{12}\) and \(L_{21}\) between the two coils are equal. From the test in Figure 1(a),
\[
\begin{align*}
V_1 &= I_1\sqrt{R_1^2 + (\omega L_1)^2} = I_1\sqrt{R_1^2 + X_1^2} \\
V_2 &= \omega L_{21}I_1 = X_{21}I_1
\end{align*}
\]
(11) (12)

The resistance \(R_1\) can be measured with an ohmmeter. Given the rms voltages \(V_1\) and \(V_2\) and the rms current \(I_1\), both \(X_1 = \omega L_1\) and \(X_{21} = \omega L_{21}\) can be determined. Similarly, from the test in Figure 1(b), both \(X_2 = \omega L_2\) and \(X_{12} = \omega L_{12}\) can be determined. It can then be verified that the mutual reactances \(X_{12} = X_{21} = X_M\) are equal and so are the mutual inductances \(L_{12} = L_{21} = M\). The final model of the transformer is shown in Figure 1(d). The coefficient of coupling \(k\) between the two windings is defined as
\[
k = \frac{M}{\sqrt{L_1L_2}} = \sqrt{\frac{M_{i1}^2}{L_1i_1} \times \frac{M_{i2}^2}{L_2i_2}} = \sqrt{\frac{N_2\phi_{m1}}{N_1(\phi_{m1} + \phi_{l1})} \times \frac{N_1\phi_{m2}}{N_2(\phi_{m2} + \phi_{l2})}}
\]
(13)

From the above relationship, it is easy to check that \(k\) is always less than unity. An alternate method of determining the mutual inductance of two coupled coils is shown in Figure 2(a). Suppose that the two coils are coupled such that they create fluxes that aid each other in the core. The total flux linkages of the series-aiding combination is
\[
\lambda_p = \lambda_1 + \lambda_2 = (L_1i_1 + M_{i1}) + (M_{i1} + L_{2i2}) = (L_1 + L_2 + 2M)i = L_pi_p
\]
(14)

where \(L_p = L_1 + L_2 + 2M\) the total inductance of this coil and \(i_p = i = i_1 = i_2\). Note that
\[
v_p = (R_1 + R_2)i_p + \frac{d\lambda_p}{dt} = (R_1 + R_2)i_p + L_p\frac{di_p}{dt}
\]
(15)
\[
\tilde{V}_p = (R_1 + R_2)\tilde{I}_p + j\omega L_p\tilde{I}_p \implies V_p = I_p\sqrt{(R_1 + R_2)^2 + (\omega L_p)^2}
\]
(16)

On the other hand, if the coils are connected so that their fluxes are opposing each other in the core, then the total flux linkages of the series-opposing combination is
\[
\lambda_n = \lambda_1 - \lambda_2 = (L_1i_1 + M_{i1}) - (M_{i1} + L_{2i2}) = (L_1 + L_2 - 2M)i = L_ni_n
\]
(17)

where \(L_n = L_1 + L_2 - 2M\) the total inductance of this coil and \(i_n = i = i_1 = -i_2\). Note that
\[
v_n = (R_1 + R_2)i_n + \frac{d\lambda_n}{dt} = (R_1 + R_2)i_n + L_n\frac{di_n}{dt}
\]
(18)
\[
\tilde{V}_n = (R_1 + R_2)\tilde{I}_n + j\omega L_n\tilde{I}_n \implies V_n = I_n\sqrt{(R_1 + R_2)^2 + (\omega L_n)^2}
\]
(19)

Measuring \(L_p\) and \(L_n\) leads to the following value of the mutual inductance \(M\):
\[
M = \frac{L_p - L_n}{4}
\]
(20)
Figure 2: (a) Series-Aiding and (b) Series-Opposing (b) Connection of two Coupled Coils

3 Equipment

- Agilent DSO5014A Digital Storage Oscilloscope
- HP/Agilent 34401A Benchtop Multimeter
- Fluke 110 or 111 True RMS Multimeter
- 12.6-V AC Power Supply and two-winding transformer

4 Procedure

1. Set up a step-down two-winding transformer with the 400-turn coil on the primary side connected to the 12.6-V AC power supply and leave the 200-turn coil open-circuited on the secondary side as shown in Figure 3(a). Turn the AC power switch ON and apply 12.6 V to the 400-turn coil. Make sure that the labels indicating the number of turns (200 and 400 turns, respectively) are showing on the top faces of each coil. Label the input primary terminals as 1-2 with terminal 1 being the “positive” terminal and terminal 2 being the negative terminal. Assume mentally that terminal 1 is marked with a dot. Similarly, label the terminals on the secondary side as 3-4. Assume that terminal 3 is the positive terminal and terminal 4 the negative one. Observe the voltages $v_{12}(t)$ and $v_{34}(t)$ on the oscilloscope and record the peak-to-peak amplitudes of both voltages as well as their phase shift. (This phase shift is either $0^\circ$ or $180^\circ$.) Deduce where the second polarity dot should be placed (terminal 3 or terminal 4).


<table>
<thead>
<tr>
<th>$V_{12,pp}$ (V)</th>
<th>$V_{34,pp}$ (V)</th>
<th>$\Delta \theta$ (deg)</th>
<th>Dotted Terminals</th>
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2. Hook up the two-winding transformer as shown in Figure 3(b). Measure and record the DC resistance of coil 1-2 using a handheld multimeter. Apply a 12.6 VAC to the primary side and record the primary rms current and the primary and secondary rms voltages. The benchtop multimeter is used to read the rms current in the primary (400-turn) winding.


<table>
<thead>
<tr>
<th>$R_1$ (Ω)</th>
<th>$I_1$ (Arms)</th>
<th>$V_{12}$ (Vrms)</th>
<th>$V_{34}$ (Vrms)</th>
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3. Turn off the AC power supply. Unhook the two-winding transformer. Measure and record the DC resistance of coil 3-4 using a handheld multimeter. Connect the secondary side to the AC power supply by applying 6.3 VAC to the secondary winding as shown in Figure 3(c). Record the secondary rms current and the secondary and primary rms voltages.

<table>
<thead>
<tr>
<th>$R_2$ ($\Omega$)</th>
<th>$I_2$ (Arms)</th>
<th>$V_{34}$ (Vrms)</th>
<th>$V_{12}$ (Vrms)</th>
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4. Connect terminals 2 and 3 of the two-winding transformer as shown in Figure 4(a). Apply 6.3 V to terminals 1 and 4. Record the series rms current in both coils, the series rms voltage of both coils, and the individual voltages across each coil.

<table>
<thead>
<tr>
<th>$I_1 = I_2$ (Arms)</th>
<th>$V_{14}$ (VAC)</th>
<th>$V_{12}$ (Vrms)</th>
<th>$V_{34}$ (Vrms)</th>
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5. Connect terminals 2 and 4 of the two-winding transformer as shown in Figure 4(b). Apply 12.6 V to terminals 1 and 3. Record the series rms current in both coils, the series rms voltage of both coils, and the individual voltages across each coil.

<table>
<thead>
<tr>
<th>$I_1 = I_2$ (Arms)</th>
<th>$V_{13}$ (Vrms)</th>
<th>$V_{12}$ (Vrms)</th>
<th>$V_{34}$ (Vrms)</th>
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Figure 3: Polarity and Turns Ratio Tests (a) and Self and Mutual Inductance Measurements (b)-(c)
5 Data Analysis and Interpretation

1. Compute the voltage ratio $v_{12}/v_{34}$ of the 400-turn and 200-turn coils in Part 1 and compare it to the theoretical value corresponding to the turns ratio $400/200 = 2$. Explain why it is greater or smaller than the theoretical value.

2. Calculate the self and mutual reactances $X_1$ and $X_{21}$ from the measurements of Part 2. Deduce the self and mutual inductances $L_1$ and $L_{21}$ in mH, respectively.

3. Calculate the self and mutual reactances $X_2$ and $X_{12}$ from the measurements of Part 3. Deduce the self and mutual inductances $L_2$ and $L_{12}$ in mH, respectively.

4. Verify that the mutual reactances $X_{12} = X_{21} = X_M$ and the mutual inductances $L_{12} = L_{21} = M$ found above are equal.

5. Compute the coefficient of coupling $k$ of the two-winding transformer from the measured values of $L_1$, $L_2$, $L_{12}$, and $L_{21}$ and check that it is less than unity.

6. Using the measurements in Parts (4) and (5), compute the inductances $L_p$ and $L_n$ of the series-aiding and series-opposing coil connections. Then compute

$$L_{12} = L_{21} = M = \frac{L_p - L_n}{4}$$

and compare this value to the previously calculated values from questions (2) and (3).
Boise State University
Department of Electrical and Computer Engineering
ECE 212L – Circuit Analysis and Design Lab
Experiment #6: Transformers, Mutual Inductance, and Coupled Coils

Date:

Data Sheet

Recorded by:

<table>
<thead>
<tr>
<th>Equipment List</th>
</tr>
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<tbody>
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<td>Equipment Description</td>
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<td>HP/Agilent 34401A Benchtop Multimeter</td>
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<td>Fluke 111 True RMS Multimeter</td>
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Part 1: Polarity Test #1:

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<th>$\Delta \theta$ (deg)</th>
<th>Dotted Terminals</th>
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<td>1 and 2</td>
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Part 2: Self and Mutual Inductance Measurement Test #1:

<table>
<thead>
<tr>
<th>$R_1$ (Ω)</th>
<th>$I_1$ (Arms)</th>
<th>$V_{12}$ (Vrms)</th>
<th>$V_{34}$ (Vrms)</th>
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Part 3: Self and Mutual Inductance Measurement Test #2:

<table>
<thead>
<tr>
<th>$R_2$ (Ω)</th>
<th>$I_2$ (Arms)</th>
<th>$V_{34}$ (Vrms)</th>
<th>$V_{12}$ (Vrms)</th>
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Part 4: Inductance Measurement of Two Series-Opposing Coupled Coils:

<table>
<thead>
<tr>
<th>$I_1 = I_2$ (Arms)</th>
<th>$V_{14}$ (VAC)</th>
<th>$V_{12}$ (Vrms)</th>
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Part 5: Inductance Measurement of Two Series-Aiding Coupled Coils:

<table>
<thead>
<tr>
<th>$I_1 = I_2$ (Arms)</th>
<th>$V_{13}$ (Vrms)</th>
<th>$V_{12}$ (Vrms)</th>
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